

# Simple techniques for measuring the base helix angle of involute gears

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**Abstract**—The paper presents two new techniques to measure the basic kinematically-relevant parameters of an involute helical gear. The techniques require taking only span and overpin measurements of the inspected gear. Among the output data of the suggested techniques is the base helix angle, a parameter that is commonly detected only by specific checking instruments. The suggested methods complement an already known procedure and, together with it, find application in classroom practice on gear geometry and whenever all basic dimensions of an involute helical gear have to be estimated without resorting to a gear inspection department. A numerical example shows application of one of the presented techniques to a case study.

**Keywords:** involute gear, gear inspection, helix angle.

## I. Introductions

The existing techniques for inspecting spur or helical involute gears cover a broad spectrum of purposes with different degrees of measurement accuracies ([1]-[3]). Most of these techniques require specific measuring instruments commonly found only in the checking departments of gear manufactures.

Although less sophisticated than other inspection techniques, the span measurement and the overpin measurement are so direct and require so simple an instrumentation that they can be taken advantage of even outside gear inspection rooms, namely, on the production floor or in the field. For the same reasons, the span and overpin measurements are also the gear measuring techniques that prove most suited to be demonstrated to engineering students and practiced by them in order to gain a better grasp of the geometry of involute gears.

As is known, all fundamental geometric parameters of an involute spur gear can be estimated by a few span measurements ([4], [5]). Unfortunately, the same does not hold for an involute helical gear, because no information about the base helix angle (or, equivalently, the lead of the involute helicoids) can be gained by span measurements only. The simplest known technique for measuring the base helix angle of an involute gear is perhaps the one that requires the inspected gear to be placed on a sine bar [6]. Nevertheless this method, together with the several others that take advantage of purposely-designed checking instruments and CNC inspection machinery, is not suited

to be employed outside gear inspection departments. A few years ago, an interesting procedure has been presented in [7] to estimate the base helix angle of external involute gears by relying on one overpin measurement and two span measurements.

This paper builds on the ideas presented in [7] by suggesting two novel procedures that complement the existent one to form a homogeneous family of techniques for measuring the base helix angle of involute gears. Common feature to all of these techniques is the need to execute three measurements of the overpin or span types. Specifically, the two new procedures presented in this paper require, respectively, two and three overpin measurements of the inspected gear.

Differently from the already known procedure, one or both of the new techniques are applicable even to thin external helical gears and also to internal spur or helical gears. In any case, based on the outcome of the three measurements, a univariate transcendental equation is obtained whose root allows the sought-for base helix angle to be determined.

Finally a numerical example shows application of one of the proposed techniques to a case study.

## II. Background information

The key parameters of an involute helical gear are the number  $z$  of teeth, the radius  $\rho$  of the base cylinder, the base helix angle  $\beta_b$ , and the base transverse tooth thickness  $s_b$ . All these parameters, with the exception of  $z$ , are shown in Fig. 1. (Regardless of the actual radial extent of the tooth flanks, the involute helicoids of a tooth are shown in Fig. 1 as extending inwards up to the base cylinder.)

Quantities such as the transverse and normal modules  $m$  and  $m_n$ , the transverse and normal pressure angles  $\alpha$  and  $\alpha_n$ , etc. ([8], [9]), do not directly characterize the gear. Rather, they refer to both the rack cutter that is ideally able to generate the gear, and the setting of such a tool relative to the gear being cut. Because infinitely many rack cutters can be used to generate the same gear, none of them should be bestowed, in principle, with the role of characterizing the gear. Despite this, and due to the existence of series of normalized normal modules [10] and commonly-selected values for the pressure angle of rack cutters, the attempt will be shown further on to single out the possible rack cutter with normalized parameters that is

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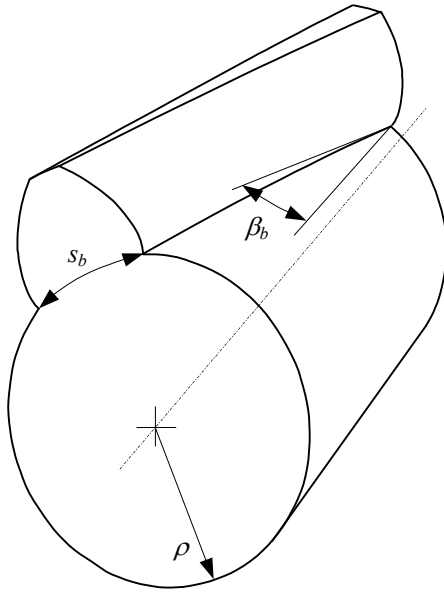


Fig. 1. Key parameters of an involute helical gear.

capable of generating an involute gear characterized by a given set of values for  $z$ ,  $\rho$ ,  $\beta_b$ , and  $s_b$ .

Additional geometric parameters such as tip diameter, root diameter, etc., will be disregarded in this paper.

#### A. The span measurement

The span (or Wildhaber) measurement  $W_k$  of an external helical involute gear as a function of the basic geometric parameters of the gear is provided by ([1]-[8])

$$W_k = (k-1)p_{bn} + s_{bn} \quad (1)$$

where  $k$  is the number of gear teeth interposed between the disc anvils of the micrometer;  $p_{bn}$  is the normal base pitch of the gear ([8]), i.e., the distance – measured on the base cylinder – between the base helices of homologous involute helicoids of two adjacent teeth (it is also the linear distance between the aforementioned involute helicoids); and  $s_{bn}$  is the base normal thickness of a tooth, i.e., the distance – measured on the base cylinder – between the two base helices of the same tooth. Quantity  $p_{bn}$  is related to the transverse base pitch  $p_b$  by the following condition ([8])

$$p_{bn} = p_b \cos \beta_b \quad (2)$$

where  $p_b$  is obviously given by

$$p_b = \frac{2\pi\rho}{z} \quad (3)$$

Similarly, the relationship between the base normal thickness  $s_{bn}$  and the transverse normal thickness  $s_b$  is

$$s_{bn} = s_b \cos \beta_b \quad (4)$$

As is known, the span measurement is feasible only if the width  $L$  of the gear satisfies the following inequality

$$L > W_k \sin \beta_b \quad (5)$$

#### B. The overpin measurement

For the measurement over pins (or balls), the distance  $e$  of the pin axes (ball centers) from the gear axis is related to the measured value  $F_a$  by the ensuing condition ([1]-[4], [8], [9])

$$e = \frac{F_a \mp a}{2 \sin \left[ \frac{\pi}{z} \text{int} \left( \frac{z}{2} \right) \right]} \quad (6)$$

where  $a$  is the diameter of both pins (balls) and  $\text{int}(\cdot)$  is the function that returns the integer part of its argument. In Eq. (6) and in the sequel of this subsection, the upper or lower sign has to be used depending on whether the gear is external or, respectively, internal.

In turn, quantity  $e$  depends both on the basic geometric parameters of the inspected gear and on the pin diameter according to the following expression

$$e = \frac{\rho}{\cos \vartheta} \quad (7)$$

In Eq. (7) angle  $\vartheta$  is implicitly provided by

$$\text{inv } \vartheta = \pm \left( \frac{s_{bn} + a}{2\rho \cos \beta_b} - \frac{\pi}{z} \right) \quad (8)$$

Function  $\text{inv}(\cdot)$  appearing in Eq. (8) is defined by

$$\text{inv } u = \tan u - u \quad (9)$$

In evaluating the right-hand side of Eq. (9), angle  $u$  has to be expressed in radians.

### III. The three measuring techniques

This section will briefly review the existing measuring technique [7] and presents the new ones. All three techniques will be explained by referring to external gears, though the last one is suitable for internal gears too.

#### A. Two span measurement and one overpin measurements

Once it has been recognized that the fundamental geometric parameters of an involute gear are four ( $z$ ,  $\rho$ ,  $\beta_b$ , and  $s_b$ , or equivalently – thanks to Eq. (4) –  $z$ ,  $\rho$ ,  $\beta_b$ , and  $s_{bn}$ ) it is a matter of course that three distinct measurements are necessary – in addition to the count of the number  $z$  of teeth – in order to identify a gear.

According to the procedure presented in [7], two span measurements have to be taken with different values of  $k$ , for instance,  $\bar{k}$  and  $\bar{k} + 1$ . These two measurements allow linear determination of quantities  $p_{bn}$  and  $s_{bn}$  ([4], see also Eqs. (1))

$$p_{bn} = W_{\bar{k}+1} - W_{\bar{k}} \quad (10)$$

$$s_{bn} = \bar{k} W_{\bar{k}} - (\bar{k} - 1) W_{\bar{k}+1} \quad (11)$$

Quantity  $s_{bn}$  is one of the four elemental parameters of the gear. As for  $p_{bn}$ , its known value is instrumental in laying down an equation in the remaining unknown parameters, i.e.,  $\rho$  and  $\beta_b$ . By merging Eqs. (2) and (3), the ensuing condition is obtained

$$\rho \cos \beta_b = \frac{z p_{bn}}{2\pi} \quad (12)$$

whose right-hand side has to be considered as known (Eq. (10)).

The last of the required measurements is the overpin measurement  $F_a$ , taken with pins of known diameter  $a$ . This measurement results in a value for parameter  $e$  (see Eq. (6)). An equation set composed of Eqs. (7), (8), and (12) can now be considered that has  $\rho$ ,  $\beta_b$ , and  $\mathcal{G}$  as unknowns. Insertion of expression (12) for  $\rho \cos \beta_b$  into Eq. (8) leads to

$$\text{inv } \mathcal{G} = \frac{\pi}{z} \left( \frac{s_{bn} + a}{p_{bn}} - 1 \right) \quad (13)$$

All quantities on the right-hand side of Eq. (13) are known *a priori* (pin or ball diameter  $a$ ), or have already been determined ( $z$ ,  $s_{bn}$ , and  $p_{bn}$ ). Therefore Eq. (13) – which has a classical form in involutometry – can be solved for unknown  $\mathcal{G}$  by a numeric iterative algorithm (bisection, Newton-Raphson, etc.).

Once the value of  $\mathcal{G}$  has been determined, Eq. (7) linearly yields the value of  $\rho$

$$\rho = e \cos \mathcal{G} \quad (14)$$

Finally the base helix angle  $\beta_b$  can be found via Eq. (12)

$$\beta_b = \cos^{-1} \left( \frac{z p_{bn}}{2\pi \rho} \right) \quad (15)$$

This concludes determination of the four basic parameters of the inspected gear, i.e.,  $z$ ,  $\rho$ ,  $\beta_b$ , and  $s_{bn}$ . If  $s_b$  is of interest instead of  $s_{bn}$ , then Eq. (4) simply allows such a replacement.

#### B. One span measurement and two overpin measurements

The first of the proposed new techniques is presented hereafter.

As soon as the outcome of one span measurements  $W_k$  and two overpin measurements  $F_{a1}$  and  $F_{a2}$  are available, it is possible to lay down the following set of six equations (see Eqs. (1), (12), (13), and (7))

$$\begin{cases} W_k = (k-1)p_{bn} + s_{bn} \\ \rho \cos \beta_b = \frac{z p_{bn}}{2\pi} \\ \text{inv } \mathcal{G}_1 = \frac{\pi}{z} \left( \frac{s_{bn} + a_1}{p_{bn}} - 1 \right) \\ \text{inv } \mathcal{G}_2 = \frac{\pi}{z} \left( \frac{s_{bn} + a_2}{p_{bn}} - 1 \right) \\ \cos \mathcal{G}_1 = \frac{\rho}{e_1} \\ \cos \mathcal{G}_2 = \frac{\rho}{e_2} \end{cases} \quad (16)$$

where quantities  $e_i$  ( $i=1,2$ ) are given by (see Eq. (6))

$$e_i = \frac{F_{ai} - a_i}{2 \sin \left[ \frac{\pi}{z} \text{int} \left( \frac{z}{2} \right) \right]} \quad (i=1,2) \quad (17)$$

In Eq. (17),  $a_1$  and  $a_2$  are the known diameters of the first and second pairs of identical pins (or balls). Equation set (16) has six unknowns, namely,  $p_{bn}$ ,  $s_{bn}$ ,  $\rho$ ,  $\beta_b$ ,  $\mathcal{G}_1$ , and  $\mathcal{G}_2$ , which can be determined as shown hereafter.

The expression of  $s_{bn}$  obtainable from the first of Eqs. (16) is first inserted into the third and fourth of Eqs. (16)

$$\frac{z}{\pi} \frac{\text{inv } \mathcal{G}_i + k}{W_k + a_i} = \frac{1}{p_{bn}} \quad (i=1,2) \quad (18)$$

By equating the left-hand sides of Eqs. (18) for  $i=1$  and  $i=2$ , the ensuing condition can be derived

$$(W_k + a_2) \text{inv } \mathcal{G}_1 - (W_k + a_1) \text{inv } \mathcal{G}_2 + \frac{\pi k}{z} (a_2 - a_1) = 0 \quad (19)$$

Now the last two of Eqs. (16) are solved for  $\mathcal{G}_1$  and  $\mathcal{G}_2$

$$\mathcal{G}_i = \cos^{-1} \frac{\rho}{e_i} \quad (i=1,2) \quad (20)$$

These expressions for  $\mathcal{G}_1$  and  $\mathcal{G}_2$  are inserted into Eq. (19)

$$\begin{aligned} & (W_k + a_2) \left( \sqrt{e_1^2 - \rho^2} - \rho \cos^{-1} \frac{\rho}{e_1} \right) \\ & - (W_k + a_1) \left( \sqrt{e_2^2 - \rho^2} - \rho \cos^{-1} \frac{\rho}{e_2} \right) \\ & + \frac{\pi k \rho}{z} (a_2 - a_1) = 0 \end{aligned} \quad (21)$$

Equation (21) contains the radius of the base cylinder,  $\rho$ , as only unknown, which can therefore be determined by resorting to a numeric algorithm for solving univariate transcendental equations. Subsequently, once  $\mathcal{G}_1$  has been computed by the first of Eqs. (20), the first of Eqs. (18)

yields the value of  $p_{bn}$ . Now the first two conditions in Eq. (16) can be solved for  $s_{bn}$  and  $\beta_b$  respectively, thus completing the identification of the geometry of the inspected gear in terms of parameters  $z$ ,  $\rho$ ,  $\beta_b$ , and  $s_{bn}$ .

### C. Three overpin measurements

The second of the proposed new techniques is explained in the following. It requires no span measurement and three overpin measurements,  $F_{a1}$ ,  $F_{a2}$ , and  $F_{a3}$ , taken by three pairs of pins (or ball) having diameters  $a_1$ ,  $a_2$ , and  $a_3$ . At first, quantities  $e_i$  ( $i=1,2,3$ ) are obtained by Eq. (6), here re-written for the sake of clarity

$$e_i = \frac{F_{ai} - a_i}{2 \sin \left[ \frac{\pi}{z} \operatorname{int} \left( \frac{z}{2} \right) \right]} \quad (i = 1, 2, 3) \quad (22)$$

Rearrangement of Eq. (8) leads to

$$\left( \operatorname{inv} \mathcal{G}_i + \frac{\pi}{z} \right) 2\rho \cos \beta_b - s_{bn} - a_i = 0 \quad (i = 1, 2, 3) \quad (23)$$

If re-written in vector form, Eq. (23) becomes

$$\mathbf{M} \begin{bmatrix} 2\rho \cos \beta_b \\ s_{bn} \\ 1 \end{bmatrix} = \mathbf{0} \quad (24)$$

where matrix  $\mathbf{M}$  has the ensuing expression

$$\mathbf{M} = \begin{bmatrix} \operatorname{inv} \mathcal{G}_1 + \frac{\pi}{z} & -1 & -a_1 \\ \operatorname{inv} \mathcal{G}_2 + \frac{\pi}{z} & -1 & -a_2 \\ \operatorname{inv} \mathcal{G}_3 + \frac{\pi}{z} & -1 & -a_3 \end{bmatrix} \quad (25)$$

Equation (24) can be regarded as a homogeneous linear set of three equations that have the components of vector  $(2\rho \cos \beta_b, s_{bn}, 1)^T$  as unknowns. Since such a vector cannot vanish – its last component is unitary – matrix  $\mathbf{M}$  has to be singular, which implies the ensuing condition

$$\det \mathbf{M} = 0 \quad (26)$$

By considering Eq. (25), Eq. (26) translates into

$$(a_3 - a_2) \operatorname{inv} \mathcal{G}_1 + (a_1 - a_3) \operatorname{inv} \mathcal{G}_2 + (a_2 - a_1) \operatorname{inv} \mathcal{G}_3 = 0 \quad (27)$$

At this point, the relations that corresponds to Eq. (7) are considered

$$\mathcal{G}_i = \cos^{-1} \frac{\rho}{e_i} \quad (i = 1, 2, 3) \quad (28)$$

By inserting expressions (28) for  $\mathcal{G}_i$  ( $i=1,2,3$ ) into Eq.

(27), an equations that has the radius of the base cylinder,  $\rho$ , as the only unknown is obtained

$$\begin{aligned} & (a_3 - a_2) \left( \sqrt{e_1^2 - \rho^2} - \rho \cos^{-1} \frac{\rho}{e_1} \right) \\ & + (a_1 - a_3) \left( \sqrt{e_2^2 - \rho^2} - \rho \cos^{-1} \frac{\rho}{e_2} \right) \\ & + (a_2 - a_1) \left( \sqrt{e_3^2 - \rho^2} - \rho \cos^{-1} \frac{\rho}{e_3} \right) = 0 \end{aligned} \quad (29)$$

Once the value of  $\rho$  that satisfies Eq. (29) has been found by a numeric algorithm, its replacement into Eq. (28) leads to determination of angles  $\mathcal{G}_i$  ( $i=1,2,3$ ). Now  $\mathbf{M}$  (Eq. (25)) can be considered as a singular matrix composed of numbers (see Eq. (26)), and Eq. (24) can be regarded as an over-constrained set of three non-homogeneous linearly-dependent equations in two unknowns, namely, quantities  $2\rho \cos \beta_b$  and  $s_{bn}$ . These can be linearly determined by considering only two of the three conditions embedded into Eq. (24). As soon as the value of  $2\rho \cos \beta_b$  has been found, determining  $\beta_b$  is a trivial task ( $\rho$  is already known). This concludes identification of the basic geometry of the inspected helical involute gear in terms of parameters  $z$ ,  $\rho$ ,  $\beta_b$ , and  $s_{bn}$ .

Differently from the previous two techniques, this last technique is also applicable to external helical gear whose width is too small for Eq. (5) to be satisfied for one or two values of  $k$  (second and first technique respectively). Moreover, it is the only technique out of the three dealt with in this paper that can be applied to internal helical involute gears, provided that balls are used instead of rollers and Eqs. (22), (23), and (25) are suitably modified to incorporate the choice of the lower signs in Eqs. (6) and (8).

## IV. Remarks

### A. Determining the customary gear parameters

A few comment are provided hereafter about some auxiliary parameters commonly mentioned while describing the geometry of a gear. These parameters do not add on to the information yielded by the four basic parameters previously determined, i.e.,  $z$ ,  $\rho$ ,  $\beta_b$ , and  $s_{bn}$ . Rather, they translate the already-available knowledge about the geometry of the inspected gear into data on the geometry and the setting of the rack cutter that has (or might have) been used to generate the gear.

Any involute gear has in common the normal base pitch,  $p_{bn}$ , with any other gear or cutting tool – hob, pinion cutter, rack cutter, etc. – able to mesh with it. For the inspected gear, quantity  $p_{bn}$  can be computed by first determining  $p_b$  via Eq. (3) and inserting its value into Eq. (2). In turn,  $p_{bn}$  depends on the pitch,  $\pi m_n$ , of the rack

cutter that is potentially able to generate the gear ([8])

$$p_{bn} = \pi m_n \cos \alpha_n \quad (30)$$

Quantities  $m_n$  and  $\alpha_n$  appearing in Eq. (30) are, as is known, the module and the pressure angle of the rack cutter. Despite the existence of infinite pairs of values for  $m_n$  and  $\alpha_n$  that satisfy Eq. (30), there is the chance that the inspected gear has been obtained by a cutter that has normalized dimensions. Should this be the case,  $m_n$  and  $\alpha_n$  can be identified after a few attempts (generally is  $\alpha_n=20^\circ$ , less frequently-used values being  $14.5^\circ$ ,  $15^\circ$ ,  $17^\circ$ ,  $22.5^\circ$ ,  $25^\circ$ , and  $30^\circ$ ; the series of normalized values of  $m_n$  is traceable in standardization tables [10]).

Once  $m_n$  and  $\alpha_n$  have been singled out, it is possible to complement the auxiliary information on the inspected gear by computing the two setting parameters of the rack cutter with respect to the gear, namely, the cutting helix angle  $\beta$  and the profile shift  $x$ . The former is the angle between the generators of the teeth of the rack cutter and the axis of the gear, whereas the latter is the distance (with sign) between two parallel planes fixed to the rack cutter: one is the reference plane of the rack (i.e., the plane on which the thickness of the rack teeth matches the width of the rack tooth spaces), whereas the other is the plane that rolls – while the gear is being generated by the rack cutter – on an ideal cylinder coaxial with the gear.

Angle  $\beta$  is implicitly given by ([8])

$$\sin \beta = \frac{\sin \beta_b}{\cos \alpha_n} \quad (31)$$

The profile shift  $x$  can be obtained by ([8])

$$x = \frac{\frac{s_{bn}}{\cos \beta_b} - m \cos \alpha \left( \frac{\pi}{2} + z \operatorname{inv} \alpha \right)}{2 \sin \alpha} \quad (32)$$

where  $m$  (the transverse module) and  $\alpha$  (the so-called transverse pressure angle) are given by ([8])

$$m = \frac{m_n}{\cos \beta} \quad (33)$$

$$\sin \alpha = \frac{\sin \alpha_n}{\cos \beta_b} \quad (34)$$

At this point it is left to the reader the final choice on whether to identify a helical involute gear by four parameters ( $z$ ,  $\rho$ ,  $\beta_b$ ,  $s_{bn}$ ) or five parameters ( $z$ ,  $m_n$ ,  $\alpha_n$ ,  $\beta$ ,  $x$ ). In favor of the four-parameter choice is the one-to-one correspondence between sets of parameters and gears, whereas infinitely-many five-parameter sets identify the same gear. The only real advantage of the five-parameter choice is the explicit reference to the geometry and setting of a rack cutter by which the gear can be obtained. Although such a rack cutter is one out of the infinitely-

many that are apt to generate the gear, most frequently it is also the only one characterized by normalized values of  $m_n$  and  $\alpha_n$ .

### B. Accuracy issues and limitations

The three techniques described in section III all lay on rigorous grounds and would result in the same outcome if applied to a perfect helical involute gear. Despite this, in practice the accuracy of their results tends to decrease from the first to the third technique. The reason is mainly due to potential ill-conditioning of the set of equations to be solved: if two overpin measurements of the same gear were executed with two pairs of pins whose diameters differs only slightly, the minuscule geometric difference detectable, in principle, by the two measurements would be almost completely masked by the finite accuracy that affects the adopted measuring instrument, so that inaccurate results would be obtained. Therefore, if two or more overpin measurements of a gear are required, care must be paid in selecting as far apart from each other as possible the diameters of different pairs of pins, still preserving the correct contact between the pins and the portion of involute helicoids on the tooth flanks (to this end, the pins with the smallest diameter could be flattened in order to make them reach the inner part of the involute helicoids while avoiding contact with the root cylinder of the gear). As a rule, if more than one technique is applicable to the case at hand, preference should be granted to the technique that requires the lowest number of overpin measurements.

All three methods described in the previous section are expected to show a declining accuracy as the base helix angle  $\beta_b$  approaches zero. Should this occur, a small error affecting the argument of the  $\cos^{-1}$  function on the right-hand side of Eq. (15) would translate into a great uncertainty about the  $\cos^{-1}$  value. Fortunately, involute helical gears that are almost spur gears are quite uncommon, and the limitation just outlined is seldom relevant.

In any case, even in the most favourable conditions, the accuracy associated to the value of  $\beta_b$  assessed by any of the explained procedures is no match for the higher accuracy attainable by state-of-the-art gear inspection apparatuses. These are equipped with measuring devices that are more precise than 0.01 mm-accurate micrometers, and can make their feeler run along the full span  $L$  of the gear width in order to accurately determine the lead of the helicoidal tooth flanks. Nevertheless the proposed inspection method proves valuable whenever special checking equipment is unavailable or unjustified.

The three techniques dealt with in this paper are not applicable to spur gears ( $\beta_b = 0$ ). In case the radius  $\rho$  of the base cylinder and the base tooth thickness  $s_b$  of a spur gear have to be assessed, the three presented techniques should be modified so as to drop one overpin measurement from each of them. For instance, the three

span measurement technique reported in [7] would transform into the the span measurement procedure explained in [4] and [5] for determining parameters  $\rho$  and  $s_{bn}$  (see Eqs. (10)-(12)). Detailed description of how the remaining two techniques should be specialized for spur gears is beyond the scope of this paper.

## V. Numerical example

Application of the third procedure explained in section III is here shown with reference to an external involute helical gear that, upon inspection, has provided the following measured data

$$\begin{aligned} z &= 31 \\ F_3 &= 89.91 \text{ mm} \\ F_{4.5} &= 95.57 \text{ mm} \\ F_6 &= 100.56 \text{ mm} \end{aligned} \quad (35)$$

It is hypothesized that the three measurements over balls,  $F_3$ ,  $F_{4.5}$ , and  $F_6$  are taken by standard handheld micrometers, whose accuracy is usually 0.01 mm. The three pairs of balls are characterized by the following diameters:  $a_1=3.000$  mm,  $a_2=4.500$  mm, and  $a_3=6.000$  mm.

The values of  $e_i$  ( $i=1,2,3$ ) provided by Eq. (22) are

$$\begin{aligned} e_1 &= 43.5108 \text{ mm} \\ e_2 &= 45.5935 \text{ mm} \\ e_3 &= 47.3408 \text{ mm} \end{aligned} \quad (36)$$

(Irrespective of the accuracy affecting the input data, intermediate and final results are here expressed by four decimal digits in order to allow the reader to thoroughly check the reported computations.)

Equation (29) is satisfied by the following root

$$\rho = 40.7512 \text{ mm} \quad (37)$$

Finally, linear solution of equation set (24) leads to the remaining two basic parameters of the considered gear

$$\beta_b = 26.5641 \text{ deg} \quad (38)$$

$$s_{bn} = 5.5636 \text{ mm} \quad (39)$$

As already pointed out, the value of  $z$  (Eq. (35)), together with the values of parameters  $\rho$ ,  $\beta_b$ , and  $s_{bn}$  (Eqs. (37)-(39)) completely define the basic geometry of the inspected gear (i.e., the shape of the involute helicoids and their spacing around the base cylinder). Alternatively, parameter  $s_{bn}$  can be replaced by parameter  $s_b$  (see Eq. (4))

$$s_b = 6.2202 \text{ mm} \quad (40)$$

Now the attempt will be made of estimating the ancillary parameters  $m_n$ ,  $\alpha_n$ ,  $\beta$ , and  $x$ . For the value of  $p_{bn}$  provided by Eqs. (3) and (2), by also presuming  $\alpha_n = 20^\circ$ , Eq. (30) yields

$$m_n = 2.5025 \text{ mm} \quad (41)$$

which is very close to the normalized value  $m_n = 2.5$  mm ([10]). It is therefore reasonable to consider the inspected gear as characterized by  $\alpha_n = 20^\circ$  and  $m_n = 2.5$  mm.

Based on Eq. (31), the value of  $\beta$  is determined

$$\beta = 28.4179^\circ \quad (42)$$

Finally, Eq. (32) yields the value of the profile shift  $x$

$$x = 0.4464 \text{ mm} \quad (43)$$

## VI. Conclusion

The paper has presented two new and easy-to-implement methods applicable to a helical involute gear for measuring its basic geometric parameters and, among these, the base helix angle. Similarly to an already known procedure, the proposed methods require taking two or three measurements over pins, together with one or no span measurement respectively. Common micrometers with standard and disc anvils, together with pairs of calibrated rollers or balls, are the only needed inspection equipment.

Differently from the already known method, one of the proposed methods can be applied to internal helical involute gears too.

A numerical example has shown application of one of the presented techniques to a case study.

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